Optimal Groundwater Extraction and Water Recycling

Problem and Research Objectives

Water scarcity has long been an important issue in many regions around the world and the threat of climate change has recently brought it further to the forefront of policy discussions. On Oahu (Hawaii), for example, some experts believe that the island's groundwater resources will be "committed" within the next twenty or thirty years (Wilson Okamoto Corp. 2008). However such studies are based on water-use projections and estimates of sustainable yield that often do not consider efficient demand conservation or improved supply-side management. Management strategies such as expansion of reservoirs, implementation of wastewater recycling, improved conjunctive use of groundwater and surface water, new pricing structures, voluntary or mandatory quantity restrictions, and watershed conservation all serve to slow the onset of groundwater scarcity as demand for water continues to grow.

Analyses in the engineering literature have begun to incorporate a wide range of management instruments, such that a portfolio of demand- and supply-side strategies are chosen in conjunction with groundwater extraction (Jenkins et al. 2001, Draper et al. 2003, Jenkins et al. 2004, Wilkinson and Groves 2006). However these studies do not attempt to solve for the vectors of management instruments that maximize the present value to the users. That task requires the application of public and resource economics.

At the same time little attention has been paid in the resource-economics literature to recycled wastewater and its potential role as an intermediate or sector-specific backstop. Since each demand sector requires a particular quality of water, recycled wastewater can potentially serve as a sector-specific backstop resource. Our research addresses the problem of integrating a water-recycling program with groundwater management in an economic-welfare-maximizing framework. To investigate the principle of optimal groundwater extraction in conjunction with wastewater recycling and seawater desalination in a two-sector (household/potable and non-household/non-potable) framework we develop a dynamic groundwater economics model. For identical satellite treatment facilities around the island, i.e., a constant amortized unit cost of recycled water, we numerically solve the optimization model. The simulation results provide optimal groundwater extraction, wastewater recycling, and seawater desalination quantities in every future period; the trajectories of the aquifer-head level; the optimal implementation of groundwater substitution; and the price paths that would induce efficient groundwater use.

Methodology

The Model

We develop and solve an optimal control model that integrates groundwater extraction, water recycling, and desalination for consumption in two demand sectors. Since water demand is multifaceted, from a long-term perspective, infrastructure choice should match the varying characteristics of water required for different end-users in terms of quantity and quality.

Generally benefits from certain uses may vary by input-water quality so that optimality would not always require obtaining the minimum allowable quality for each use. To keep the model tractable, however, we aggregate non-potable uses into a single demand category and assume no additional benefit from using a higher quality source than necessary. Groundwater extraction is the primary source of high quality (potable) water. No surface water is available, but lower-quality (non-potable) water can be obtained from recycling wastewater. Desalinated seawater serves as a high-quality backstop. Potable water can be utilized as a supply source for both sectors but recycled water cannot supply high-quality demands.

The coastal groundwater aquifer is modeled as a renewable and replaceable resource, characterized by a lens of freshwater floating on underlying seawater. The head level (h), or the distance between the top of the lens and mean sea level, serves as a measure of the aquifer stock. As the freshwater stock declines the head level falls and extraction becomes more costly since water must be lifted further to reach the ground surface. Thus, unit extraction cost is a non-negative, decreasing, convex function of head, i.e. $c_G(h) \geq 0$, $c_G'(h) < 0$, and $c_G''(h) \geq 0$.

Leakage from a coastal aquifer is also a function the head level—as the head level declines leakage decreases both because of a smaller surface area along the ocean boundary and because of the decrease in pressure due to shrinking of the lens. Thus, leakage is a positive, increasing, convex function of head, i.e. $l(h) \geq 0$, $l'(h) > 0$, and $l''(h) \geq 0$. We assume that infiltration from precipitation and adjacent water bodies is fixed at a constant rate (I). The resource is "renewable," therefore, in that *net recharge* (recharge minus leakage) varies with the groundwater stock.

For satellite recycling facilities developed near concentrated groups of users (e.g., industrial parks), the timing of distribution-infrastructure construction coincides with that of the treatment facility. Given sufficient data on infrastructure (e.g., connections, pipes, and treatment plants) and variable costs, amortization methods can be applied to determine a constant unit cost of recycled water (c_R).

The water manager chooses the rates of groundwater extraction for use in the household sector (q_t^{GH}) and the non-household or "other" sector (q_t^{GO}), the rates of seawater desalination for household (q_t^{BH}) and non-household use (q_t^{BO}), and the rate of water recycling (q_t^{RO}) for use in the non-household sector to maximize the present value of net social benefit, measured as the sum of consumer and revenue surplus or equivalently gross consumer surplus less total costs, i.e.

$$\max_{q_t^{GH},q_t^{BH},q_t^{GO},q_t^{RO},q_t^{BO}} \int_0^\infty e^{-\delta t} \left\{ \begin{array}{l} \int_0^{q_t^{GH}+q_t^{BH}} D_H^{-1}(x,t)\,dx + \int_0^{q_t^{GO}+q_t^{RO}+q_b^{BO}} D_O^{-1}(x,t)\,dx - \\ \left(q_t^{GH}+q_t^{GO}\right)c_G\left(h_t\right) - \left(q_t^{BH}+q_t^{BO}\right)c_B - q_t^{RO}c_R \end{array} \right\} dt \qquad (1)$$

subject to $\gamma \dot{h}_t = I - l(h_t) - \left(q_t^{GH} + q_t^{GO}\right)$

where $D_t^{-1}(\bullet)$ is the inverse demand function for sector $i=H,O$, δ is the positive discount rate, c_B is the unit cost of desalinated seawater, c_R is the unit treatment and distribution cost of recycled wastewater, and γ is a height-to-volume conversion factor.

In solving the model (Eq. 1) we find that the objective of welfare maximization can be achieved by using the available resources in reverse order of their marginal opportunity costs (MOC) within each sector. In other words, the pricing rule that induces optimal consumption in both sectors is

$$\begin{aligned} p_t^H &= \min\left\{c_G\left(h_t\right) + \lambda_t, c_B\right\} \\ p_t^O &= \min\left\{c_G\left(h_t\right) + \lambda_t, c_R, c_B\right\} \end{aligned} \tag{2}$$

where p_t^i is defined as the marginal benefit of water consumed in sector i, and λ_t is the shadow price or marginal user cost of groundwater. While the MOCs of recycled and desalinated water are constant, the MOC of groundwater is variable, since unit extraction cost rises as the aquifer head level declines and the marginal user cost rises as the resource becomes scarcer. The stages of resource uses leading up to the long-run equilibrium or steady state will be determined by the ordering of the three MOCs (Eq. 2), as well as the shape of the variable groundwater MOC path over time.

Numerical Computations

The model is solved numerically by iterating forward through time. Growing demand in both sectors ensures eventual implementation of desalination and recycling, since the size of the aquifer is finite. The initial head level is given by field measurements, and the terminal head level is obtained from steady-state calculations. Given these boundary conditions the optimal paths will be determined once we know the correct initial shadow price of groundwater. Using a shooting method, we start with an educated guess for the initial shadow price. Governing equations for the head level and price allow one to characterize the entire paths of water use and aquifer stock. If one or more of the resulting terminal values is inconsistent with the optimal steady state conditions, then the initial guess for the shadow price must be adjusted and the process repeated. The trajectory is optimal once all of the boundary conditions are simultaneously satisfied.

Principle Findings and Significance

Analytical Results

There are several possible orderings of resource use, depending on the MOC paths in Eq. 2. If groundwater is sufficiently abundant that the initial marginal opportunity cost of groundwater (MOC^G) lies below both the unit costs of desalination and recycling, then stage 1 is characterized by exclusive groundwater use in both sectors (scenario A). The high rate of combined extraction raises MOC^G, and recycled water is implemented as a sector-specific backstop for the non-household sector in period T_1. Groundwater continues to supply the household sector in stage 2 but all non-household needs are met by recycled water. At the second switch-point (T_2), MOC^G reaches the unit cost of desalination, and the system reaches a steady state. Desalinated water supplements groundwater in the household sector and recycled water is used exclusively by the non-household sector in the steady state.

If the aquifer has already been depleted, such that $MOC^G > c_B > c_R$ in the initial period, then both "backstops" are used at the outset to allow the aquifer to replenish in stage 1 (scenario B). Once the head level rises enough to allow MOC^G to decline to c_B, extraction of groundwater commences. Any extraction greater than net recharge, however, would raise MOC^G above the unit cost of desalination and would thus be non-optimal. That is, it would be more cost-effective to use desalinated water in place of those last units of groundwater.

Thus, the system immediately reaches a steady state in which extraction is exactly equal to aquifer inflow, and the optimal quantity of water demanded for the non-household sector is met entirely by recycling.

A third possibility is that $c_B > MOC^G > c_R$. In that scenario (C), groundwater is extracted for the household sector at the outset, but recycled water is used exclusively in every period for the non-household sector. The MOC of groundwater eventually rises to the unit cost of desalination at which point the aquifer reaches a steady state. Extraction is limited to net recharge so that the head level is maintained and the remainder of the quantity demanded by the household sector is supplied by desalination. The stages of each scenario are summarized in Table 1 and hypothetical MOC paths for scenario A are illustrated in Figure 1.

Table 1. Stages of resource use with constant unit recycling cost.

Scenario/Stage	1	2	3
A	GW for H GW for O	GW for H RW for O	GW + DW for H RW for O
B	DW for H RW for O	DW + GW for H RW for O	
C	GW for H RW for O	GW + DW for H RW for O	

Note: GW = groundwater, DW = desalinated water, RW = recycled water, H = household sector, O = non-household sector

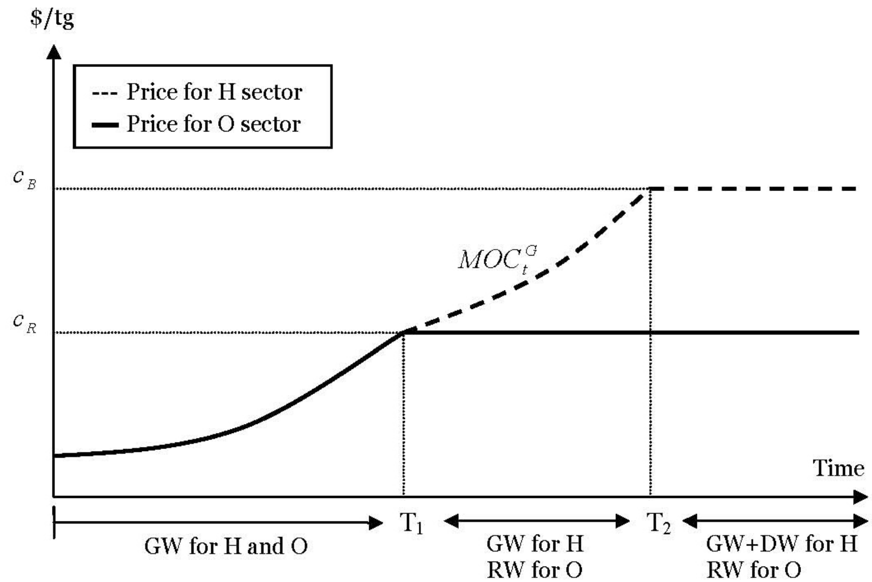

Figure 1. Hypothetical MOC paths for scenario A.

Empirical Results

In an application to the Pearl Harbor aquifer and consumption district on the island of Oahu, Hawaii, we characterize the paths of resource use when recycling is implemented optimally, implemented non-optimally or "prematurely" from the outset, and never implemented (Figure 2). The extraction structure for the optimal solution is consistent with the paths described in hypothetical scenario A. Since Pearl Harbor aquifer is currently above its optimal steady state, the scarcity value of groundwater starts low, and groundwater is initially used in both sectors. In year 75 the scarcity value reaches c_R, and the non-household sector shifts to the use of recycled water. The household sector continues to use groundwater exclusively for the next decade, and the price of groundwater continues to rise. At year 85 the system reaches a steady state—groundwater extraction is limited to net recharge, and the remaining optimal quantity demanded by the household sector is supplied by desalination.

If instead recycled water is utilized by the non-household sector from the outset, the aquifer is allowed to build up much higher than in the optimal case. Extraction is relatively low for the first 80 years because the non-household sector never uses groundwater. Total consumption is also lower for the first 60 years because, although groundwater is less scarce for the household sector, the high recycled water price for the non-household sector induces excessive conservation. When groundwater is optimized without consideration of water recycling, extraction is lower, total consumption is lower, and the scarcity value of groundwater is higher in every period.

The welfare gains from implementing an optimal water recycling program can be substantial. In this particular case study the net present value of social benefits (NPV) derived from the optimal program is approximately $1.173 billion. When groundwater is optimized alone the NPV is $1.166 billion, a difference of $67 million in present-value terms.

Recycling increases welfare by prolonging the use of groundwater before the steady state and thus delays the implementation of costly desalination for the household sector. Alternatively, if recycling is implemented prematurely, the NPV is $1.123 billion, nearly $500 million less than in the optimal case. Although early recycling delays desalination by 6 years, the NPV is still 4.5% lower because costly recycled water is used too soon. Water recycling has the potential to increase total welfare, but economic optimality may entail delaying implementation, depending on how the marginal opportunity cost of recycled water compares to that of groundwater.

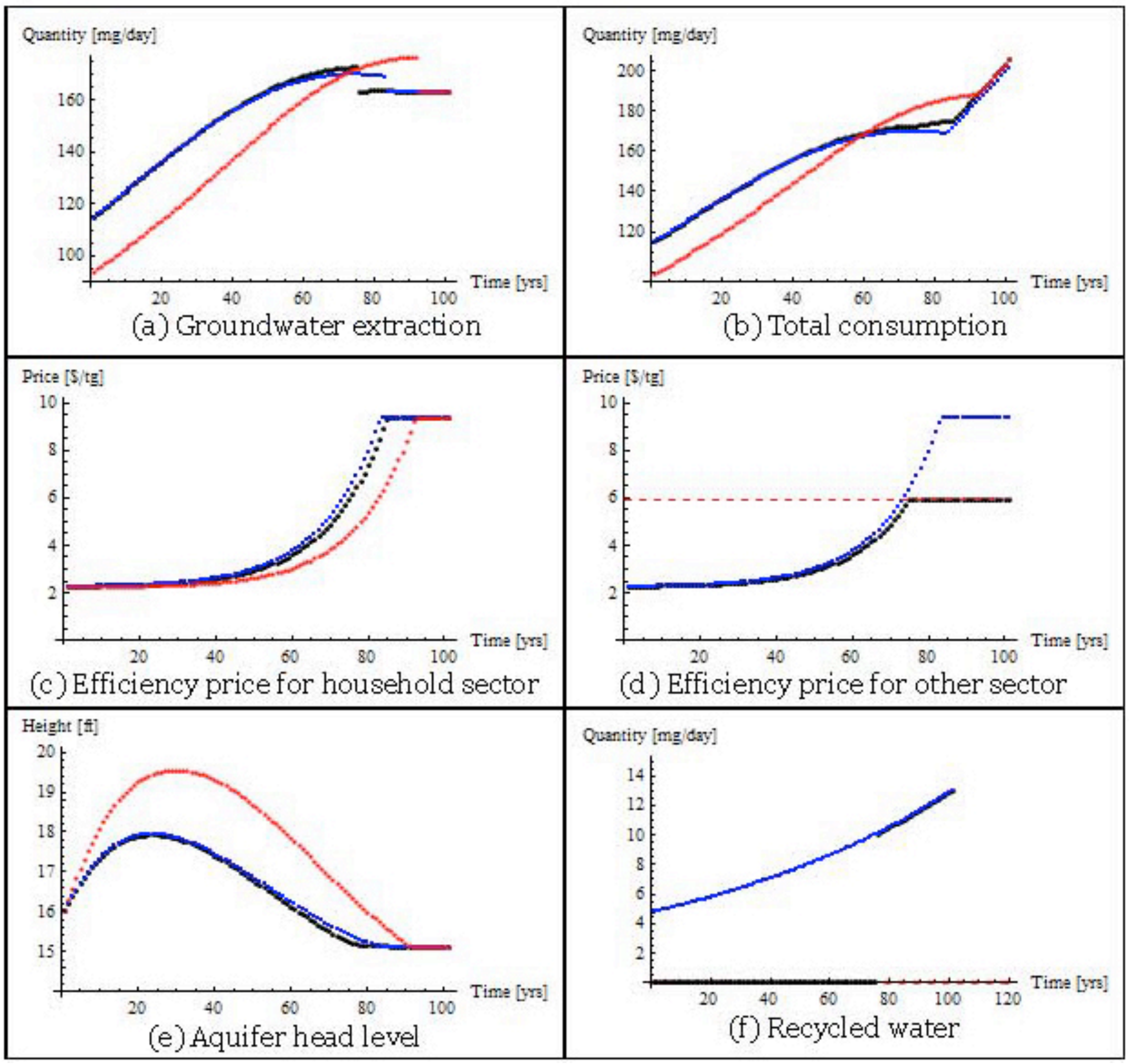

Figure 2. Various time paths for three recycling scenarios: optimal (black), none (blue), premature (red).

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